$d_m \ldots d_3\ d_2\ d_1\ d_0$ d_0 is the digit in the one's place, d_1 is in the 10's place… and so on
$x = d_0 + d_1*10 + d_2*100 + \ldots + d_m*10^m$
Most of the divisibility tests and their proofs come from examining this expression mod n for some n, and replacing the various powers of 10 with their equivalents mod n:

- Proof of Test for Divisibility by 2. Observe that 10 divided by 2 has a remainder of 0. So 10 = 0 (mod 2). Then $10^k=0^k=0$(mod 2) for k is an positive integer Hence,
  $X = d_0 + d_1*0 + d_2*0 + \ldots + d_m*0 = d_0$ (mod 2)
  Therefore, x is divisible by 2 if and only if its last digit a_0 id divisible by 2, which happens if and only if the last digits is one of 0,2,4,6,8
- Proof of Test for Divisibility by 4. Observe that 100 divided by 4 has a remainder of 0. So 100 = 0(mod4). Hence, $10^k=0$(mod 4) for k>= 2. Then,
  $X = d_0 + d_1*10 + d_2*0 + \ldots + d_m*0 = d_0 + d_1*10$(mod4)
  Therefore, x is divisible by 4 if and only if the number $a_0 + a_1*10$ is divisible by 4. But $a_0 + a_1*10$ is the number formed by keeping only the last two digits of x. So x is divisible by 4 if and only if the number formed by dropping all but the last two digits of x.
- Proof of Test for Divisibility by 9. Observe that 10 divided by 9 has a remainder of 1. So 10 = 1(mod 9). Hence $10^k=1^k=1$(mod9) for k is a positive integer. Then,
  $X = d_0 + d_1*1 + d_2*1 + \ldots + d_m*1 = d_0 + d_1 + d_2 + \ldots + d_m$ (mod9)
  Therefore, x is divisible by 9 f and only if the sum of its digits is divisible by 9.
  10^k = 99..9 + 1 with k amount of 9.
  $10^k = (d_{k-1} * 10^{k-1}) + (d_{k-2} * 10^{k-2})+\ldots+ (d_0 * 10^0)+1$
- Proof of Test for Divisibility by 11. Observe that 10 = -1 (mod 11).
  Hence, $10^k = (-1)^k$(mod 11) for k is an positive integer. Then,
  $X = d_0 + d_1*(-1) + d_2*(-1)^2 + \ldots + d_m*(-1)^m = d_0 - d_1 + d_2 - d_3 + \ldots + d_m(-1)^m$(mod11)
  Therefore, x is divisible by 11 if and only if alternating sum of its digits $d_0-d_1+d_2-d_3+\ldots+d_m(-1)^m$ is divisible by 11.
- Proof of Test for Divisibility by 12. Since $12 = 2^2 3$ involves more than one prime, lets start a different way. If a number is divisible by 12, then in its prime factorization it must contain $2^2 3$, possibly along with other prime factors (perhaps even some more 2s and 3s). Therefore, the number must be divisible by both 3 and 4. Therefore, a number is divisible by 12 if and only if it is divisible by both 3 and 4

a) The product of two consecutive integers is even.
b) Two consecutive integers always have opposite parity.
c) The product of an even integer and any other integer is even.
d) The sum of an even and odd integer is odd.
e) The product of two odd integers is odd.
f) The product of even integers is even.
g) The sum odd two even integers is even.

1. Square of even is even.
2. Square of odd is odd.
3. Odd plus even is odd.
4. Odd + 1 is even.

Tautology: always true
Contradiction: always false
Bi-conditional: p←→ q

"P is sufficient for q" means (p->q) P=T q=T/F
"p is necessary or Q" means (-p->-q)=q->p

	Logical Equivalences	Set Properties
	For all statement variables p, q, and r:	For all sets A, B, and C:
Commutative	$p \lor q \equiv q \lor p$ $p \land q \equiv q \land p$	$A \cup B = B \cup A$ $A \cap B = B \cap A$
Associative Distributive	$p \land (q \land r) \equiv p \land (q \land r)$ $p \lor (q \lor r) \equiv p \lor (q \lor r)$	$A \cup (B \cup C) = A \cup (B \cup C)$ $A \cap (B \cap C) = A \cap (B \cap C)$
	$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$ $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
	$p \lor \mathbf{c} \equiv p$ $p \land \mathbf{t} \equiv p$	$A \cup \varnothing = A$ $A \cap U = A$
Negation	$p \lor \sim p \equiv \mathbf{t}$ $p \land \sim p \equiv \mathbf{c}$	$A \cup A^c = U$ $A \cap A^c = \varnothing$
Double negation	$\sim(\sim p) \equiv p$	$(A^c)^c = A$
Idempotent	$p \lor p \equiv p$ $p \land p \equiv p$	$A \cup A = A$ $A \cap A = A$
Universal Bound	$p \lor \mathbf{t} \equiv \mathbf{t}$ $p \land \mathbf{c} \equiv \mathbf{c}$	$A \cup U = U$ $A \cap \varnothing = \varnothing$
Demorgan's	$\sim(p \lor q) \equiv \sim p \land \sim q$ $\sim(p \land q) \equiv \sim p \lor \sim q$	$(A \cup B)^c = A^c \cap B^c$ $(A \cap B)^c = A^c \cup B^c$
Absorption	$p \lor (p \land q) \equiv p$ $p \land (p \lor q) \equiv p$	$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$
	$\sim \mathbf{t} \equiv \mathbf{c}$ $\sim \mathbf{c} \equiv \mathbf{t}$	$U^c = \varnothing$ $\varnothing^c = U$

Set Properties

- Inclusion of Intersection:

  $A \cap B \subseteq A$ and $A \cap B \subseteq B$
- Inclusion in Union:

  $A \subseteq A \cup B$ and $B \subseteq A \cup B$
- Transitive Property of Subsets:

  $A \subseteq B$ and $B \subseteq C \rightarrow A \subseteq C$
- $x \in A \cup B \Leftrightarrow x \in A$ or $x \in B$
- $x \in A \cap B \Leftrightarrow x \in A$ and $x \in B$
- $x \in B - A \Leftrightarrow x \in B$ and $x \notin A$
- $x \in A^c \Leftrightarrow x \notin A$
- $(x, y) \in A \times B \Leftrightarrow x \in A$ and $y \in B$

n is prime if:

- $n>1$
- $\forall r,s > 0$, if $n=rs$ then $r=n$ or $s=n$

$p \rightarrow q = \sim p \rightarrow \sim q = \sim p \lor q = \sim(p \land \sim q)$

$\frac{d}{n} \Leftrightarrow \exists$ an int k s.t. $n = dk$

Every integer is rational.
Sum of rational is rational

$\frac{a}{b} + \frac{c}{d} = \frac{ad+cb}{bd}$ rational
For all int a & b, if a and b are positive and a divides b, then $a \le b$

Quotient-Remainder Theorem
Given any integer n and positive integer d, there exist unique integers q and r such that
$n = dq + r$ and $0 \le r < d$
Ex: $54 = 52+2 = 4*13+2$ hence $q = 13$ & $r=2$

Modus Ponens	$p \implies q$ p $\therefore q$	Modus Tollens	$p \implies q$ $\sim q$ $\therefore \sim p$
Elimination	$p \vee q$ $\sim q$ $\therefore p$	Transitivity	$p \implies q$ $q \implies r$ $\therefore p \implies r$
Generalization	$p \implies p \vee q$ $q \implies p \vee q$	Specialization	$p \wedge q \implies p$ $p \wedge q \implies q$
Conjunction	p q $\therefore p \wedge q$	Contradiction Rule	$\sim p \implies F$ $\therefore p$

Definition: Divisibility

- If n and d are integers and d ≠ 0 then n **is divisible by** d if n = dk for <u>some</u> integer k.
  - d | n ⇔ ∃ an integer k such that n = dk
  - – n **is a multiple of** d
  - – d **is a factor of** n
  - – d **is a divisor of** n
  - – d **divides** n
- Notation: d | n (read "d divides n")
- E.g.: 21 is divisible by 3, 32 is a multiple of −16, 5 divides 40, 6 is a factor of 54, 7 is a factor of −7

Proof by division into cases

$p \vee q$
$p \rightarrow r$
$q \rightarrow r$
$\therefore r$

Universal Transitivity

1. ∀x P(x) → Q(x).
2. ∀x Q(x) → R(x).

∴ ∀x P(x) → R(x). [Inferred from 1, 2]

<u>Example</u> (Tarski's World)

∀x, if x is a triangle, then x is blue.

∀x, if x is blue, then x is to the right of all the squares.

∴ ∀x, if x is a triangle, then x is right of all the squares

Universal Modus Ponens

1. ∀x, if P(x) then Q(x).
2. P(a) for a particular a.

∴ Q(a). [Infer Q(a) from 1, 2]

<u>Example</u>:

∀x, if (x is even) then (its square is even).

k is even.

∴ k's square is even.

Universal Modus Tollens

1. ∀x, if P(x) then Q(x).
2. ~Q(a), for a particular a.

∴ ~P(a). [Infer ~P(a) from 1, 2]

<u>Example</u>:

∀x, if x is human then x is mortal

Zeus is not mortal.

∴ Zeus is not human.

Relation among ∀, ∃, ∧, and ∨

Let D = $\{x_1, x_2, \ldots, x_n\}$.

- $\forall x \in D, Q(x)$
  $\equiv Q(x_1) \wedge Q(x_2) \wedge \cdots \wedge Q(x_n)$
- $\exists x \in D$ such that $Q(x)$
  $\equiv Q(x_1) \vee Q(x_2) \vee \cdots \vee Q(x_n)$

∀x∈D if P(x)→Q(x)
Contrapositive:
∀x∈D if ~Q(x)→ ~P(x)
Converse:
∀x∈D if Q(x) → P(x)

Inverse:
∀x∈D if ~P(x) → ~Q(x)
Contradiction:
Assume p(x) & ~Q(x)
Show impossible

Contraposition

$\forall x, \sim Q(x) \rightarrow \sim P(x)$

Assume $\sim Q(x)$ Show $\sim P(x)$

$n^2=2^n$ $n\geq 5$ $k^2<2^k$

Prove

$(k+1)^2<2^{k+1}$

$2*k^2<2^{k+1}$

$(k+1)^2<2k^2$ ← prove

Inequality Axioms

If $x<y \Rightarrow cx < cy$ for any $c>0$

$x<y \Rightarrow cx > cy$ for any $c<0$

$x<y \Rightarrow (c+x)<(c+y)$ for any c

given $x<y$

prove $y<z$

show $x<z$

$\mathbb{N}$ ➔ Natural = 0,1,2…

$\mathbb{Z}$ ➔ Integer = …-2,-1,0,1,2,…

$\mathbb{Q}$ ➔ Rational $=\frac{a}{b}$ s.t. $b\neq 0$

$\mathbb{R}$ ➔ Real # = $\sqrt{2}$, e , pi

$\mathbb{P}$ ➔ Irrational = $\sqrt{2}, \sqrt{3}$

Use S as the set of all students, s(x) to denote that x is smart, and c(x) to denote that x is in CSE215. g(x) to represent x's GPA.

All CSE215 students are smart	$\forall x \in S, c(x) \rightarrow s(x)$
Some smart students have a GPA of less than 3.5	$\exists x \in S, s(x) \wedge g(x)<3.5$
There are students in CSE215 that have a GPA of 4	$\exists x \in S, s(x) \wedge c(x) \wedge g(x)=4$
All students with a GPA of 3.8 or more are enrolled in CSE215	$\forall x \in S, s(x) \wedge g(x)\geq 3.8 \rightarrow c(x)$

B(x) (Is x a bird?) O(x)(Is x an Ostrich?) T(x)(Is x at least 9 feet tall?) I(x)(Does x belong to me?)

No birds except ostriches are at least 9 feet tall	$\forall x, B(x) \wedge O(x) \rightarrow T(x)$
There are no birds that belong to anyone but me	$\forall x, B(x) \rightarrow I(x)$
I have no birds less than 9 feet tall	$\forall x, B(x) \wedge I(x) \rightarrow T(x)$

M and F as sets of male & female. C(x) x is crazy l(x,y) x likes y

All women are crazy? No b/c there exists women who likes men and is not crazy. Men don't have to like women back (iii)

All men are crazy	$\forall m \in M, c(m)$
Crazy people like each other	$\forall c(m), \forall c(f)\ l(c(m),c(f)) \wedge l(c(m),c(m)) \wedge l(c(f),c(f)) \wedge l(c(f),c(m))$
All women like some men	$\forall f \in F, \exists m \in M\ l(f,m)$

P to be the set of all people and T to be the set of all time instants.

$M(x_1,x_2,t)$ x_1 &x_2 got married at time t. $D(x_1,x_2,t)$ same shit.

E

very person gets married at some time

Some married people get divorced later

$\exists p \in P, \exists p_1 \in P, \exists t \in T, \exists t_1 \in T$

$M(p,p_1,t) \wedge D(p,p_1,t_1) \wedge t_1>t$

Only previously unmarried people can get married

$\forall p \in P, \exists p_1 \in P, \exists t \in T\ M(p, p_1,t)$

$\forall p \in P, \forall p_1 \in P, \exists p_2 \in P\ \exists t_1 \in T, \exists t_2 \in T$

$M(p,p_1,t) \rightarrow \sim M(p,p_2,t_2) \wedge \sim M(p_1,p_2,t_2) \wedge t_2<t_1$

M be sets of all movies.

I(m) interesting B(m) bought W(m) watched P(m)made by W.Peterson E(m) Movie have 8+ rating

I buy all interesting movies.	$\forall m \in M, I(m) \rightarrow B(m)$
I watched all movies by Wolfgang Peterson.	$\forall m \in M, P(m) \rightarrow W(m)$
For any movie, either I buy it or watch it	$\forall m \in M, B(m) \vee W(m)$
All interesting movies have a rating of more than 8.	$\forall m \in M, I(m) \rightarrow E(m)$
The movie Troy has a rating of 7.5	$\sim E(Troy)$

Troy is by wolfgang Peterson? No.

T(x) tall. B(x) basketball player

Every basketball player is tall	$\forall x, B(x) \rightarrow T(x)$
Among all basketball players, some are tall	$\exists x, B(x) \wedge T(x)$
Some of all the tall people are basketball players	$\exists x, T(x) \wedge B(x)$
Anyone who is tall is a basketball player	$\forall x, T(x) \rightarrow B(x)$
All people who are basketball players are tall	$\forall x, B(x) \rightarrow T(x)$
Anyone who is a basketball player is a tall person	$\forall x, B(x) \rightarrow T(x)$

Prove that $\sqrt{2}+\sqrt{3}$ is irrational. Assume $\sqrt{6}$ is irrational

Contradiction. $\sqrt{2}+\sqrt{3}$ is rational

Square it. $(\sqrt{2}+\sqrt{3})^2 = 5+2\sqrt{6}$ = rational because rational * rational is rational.

$5+2\sqrt{6} = r$ $\quad r/2 - 5/2 = \sqrt{6}$ $\quad$ contradiction b/c rational – rational = rational

Prove that if r^2 is irrational then r is irrational

r(x) ➔ x is rational

$\sim r(r^2) \rightarrow \sim r(r)$ $\quad r(r) \rightarrow r(r^2)$ $\quad$ if r is rational, then r^2 is rational

r=a/b s.t. b≠0 square both sides

$r^2=a^2/b^2$ $\; b^2 \neq 0$ $\quad$ thus, r^2 is rational and r is rational.

If a is rational and b is irrational, then (a+b) is irrational

$r(a) \wedge \sim r(b) \rightarrow \sim r(a+b)$ $\quad r(a) \wedge \sim r(b) \wedge r(a+b)$ $\;$ counter example

r(a) = q/r $\quad$ r(a+b)=s/t $\quad$ r≠0 $\quad$ t≠0

a+b-a=b $\quad$ b=(sr-qt)/tr

tr≠0 contradiction

If a and b are irrational, then (a+b) is irrational

False $-\sqrt{2}+\sqrt{2}=0$

If a is rational and b is irrational, then a*b is irrational

r=0 b=$\sqrt{2}$ $0+\sqrt{2}=0$ ← rational statement is false.

If $\sqrt{ab}$ is irrational, then $\sqrt{a}+\sqrt{b}$ is irrational. Contradiction

$\sqrt{ab}$ rational & $\sqrt{a}+\sqrt{b}$ rational

$\sqrt{a}+\sqrt{b}$ = x/y y!=0 square both sides. $a+\sqrt{ab}+b=x^2/y^2$

$\sqrt{ab} = (x^2-ay^2-by^2)/y^2$ $\quad y^2!=0$ b/c y!=0

contradiction. Original statement is true.

For all integers n greater than 2 prove that (n^2-1) is not prime.

$\forall n \in \mathbb{Z}, (n>2) \rightarrow (n^2-1)$ is not prime

Proof: Pick an arbitrary n and assume n>2.

$(n^2-1)=(n-1)(n+1)$

(n-1) never equals to 1 because n>2.

(n+1) can never equal (n^2-1) b/c (n-1)≠1

Thus, (n-1) and (n+1) is never equal to 1 or (n^2-1)

Prove the $\sqrt{3}$ is irrational.

Counter example: Suppose $\sqrt{3}$ is rational

$\sqrt{3}$ = a/b   b≠0 $3=a^2/b^2$ $3b^2=a^2$ $b^2\neq0$

Product of two odd integers is odd. a=2n+1   b=2m+1

$3(2m+1)^2=(2n+1)^2$ $2(3m^2+3m+1)=2n^2+2n+1$

One side is even other is odd. Contradiction.

Use Mathematical Induction to prove that $2^n<(n+1)!$ For all n≥2

Base case: n=2 4<6

Induction Hypothesis: …

Prove p(k+1): $2^{k+1}<(k+2)\neq(k+2)(k+1)!$

$2*2^k<(k+2)(k+1)!$

2< k+2 ← Since k≥2 and $2^k<(k+1)!$ Is true by IH. Statement is true.

Using the element argument, prove that if **A ∪ B = B ∩ A then A=B**

Suppose x∈A, then x∈A∪ B & x∈A∩B Since(A ∪ B = B ∩ A)

So, A∈ B, b/c x∈A∩ B we conclude A⊆B

Suppose x∈B, then x∈A∪ B & x∈A∩ B   Since(A ∪ B = B ∩ A)

So, B∈A , b/c x∈A∩ B we conclude B⊆A

Since A⊆B & B⊆A then A=B

Using the element argument, prove that **(A-C) ∩B = (A∩B)-C**

Suppose (A-C) ∩B≠(A∩B)-C   Contradiction

if x∈(A-C) ∩B, then x∈(A-C) & x∈B   x∈A & x!∈C & x∈B

x∈(A∩B) & x!∈C   x∈(A∩B)-C

Contradiction

Show **!(A∩B) = !A ∪!B**

x∈!(A∩B) → x!∈B∩!A or a∈A∩!B ← x∈!B or a∈!A∩!B ← x∈!A∩!B

a∈!A or !B = a∈!A ∪ !B !(A∩B) ⊆ !A ∪!B

Use the element argument to prove that for all sets A,B and C if **A⊆B and A⊆C then A⊆B∩C.**

Assume x∈A then by A⊆B x∈B

And by A⊆C x∈C

x∈B ^ x∈C = x∈B∩C

Thus, A⊆ B∩C

For all sets A,B,C if **B∩C⊆A then, (C-A)∩(B-A)=empty set**

Assume contradiction there is some element x in (C-A))∩(B-A)

x∈(C-A) ∩ (B-A)   x∈B∩C and x!∈A

Since B∩C⊆A and x!∈A   Contradiction.

Prove or disprove. Given any set X and given any functions *f*: X→ X, g:X→X, h:X→X, if h is one-to-one and h°f= h°g, then f = g

Let x∈ X

f(x)=g(x) ∀x∈X

h(f(x))=h(g(x))

f = g

Lets show that for all x, f(x) = g(x). Pick an arbitrary x. We know that h(f(x))=h(g(x)) since the composition functions are equal. Since h is one-to-one, the above implies that f(x)=g(x)

f°h= g°h

∀x f(h(x))=g(h(x))

f(x) = 0 when x is even and 1 when x is odd

g(x) = 0∀x

f=g only when x is even

Let *f*: ℝ→ ℝ be a function from the set of real numbers to the set of real numbers, and c a non-zero real number.

a) If *f* is one-to-one, is c*f also one-to-one?
   Let g(x) = c*f(x) ∀x
   We need to show that g is one-to-one if *f* is.
   Pick arbitrary x_1 & x_2. Assume $g(x_1)=g(x_2)$ then $c*f(x_1) = c*f(x_2)$
   Thus, $x_1=x_2$ since *f* is one-to-one.
   Thus, $g(x_1)=g(x_2)$ $x_1=x_2$. g is one-to-one so c*f also one-to-one
b) If *f* is onto, is c*f also onto?
   We need to show that g is onto if *f* is.
   Pick arbitrary y in ℝ.
   We need to discover an x s.t. g(x)=y, c.f(x)=y
   Pick an x s.t. f(x)=y/c.
   Since f is onto, such an x exists.
   Verify g(x)=c.f(x)=c.y/c=y
   Thus g is onto

Let sets R, S and T be defined as follows

R = {x∈ ℤ | x is divisible by 2}

S = {y∈ ℤ | y is divisible by 3}

T = {z∈ ℤ | z is divisible by 6}

Is T⊆R?

a⊂T

a is divisible by 6

a=6b for some into b

a=3*2*b = 2(3b)

a is divisible by 2

R is divisible by 2

A = {x∈ ℤ | x=6a+4 for some int a}

B = {y∈ ℤ | y=18b-2 for some int b}

C = {z∈ ℤ | z=18c+16 for some into c}

A⊆B?

Use counter example

A={…,-8,2,4,10,16,…}

B={…,-20,-2,16,…}

$2 \in A$ & $2 ! \in B$
$A ! \subseteq B$
$!(A \cap B) = !A \cup !B$

$Ax(B \cup C) = (AxB) \cup (AxC)$ ← Prove $\subseteq$ of each other
Let $(x,y) \in Ax(B \cup C)$ $\quad x \in A \cap y \in (B \cap C)$
Case 1: $y \in B$
$x \in A$ and $y \in B$ so $(x,y) \in (AxB)$
$(x,y) \in (AxB) \cup (AxC)$
Case 2: $y \in C$
$x \in A$ and $y \in C$ so $(x,y) \in (AxC)$
$(x,y) \in (AxC) \cup (AxB)$
therefore $Ax(B \cup C) \subseteq (AxB) \cup (AxC)$
Not let $(x,y) \in (AxB) \cup (AxC)$

Case1: $(x,y)\,) \in (AxB)$ ➔ $x \in A$ & $y \in B$
$x \in A$ & $y \in B \cup C$ ➔ $(x,y) \in Ax(B \cup C)$
Case2: $(x,y)\,) \in (AxC)$ ➔ $x \in A$ & $y \in C$
$x \in A$ & $y \in C \cup B$ ➔ $(x,y) \in Ax(C \cup B)$

$|A \cup B| + |A \cap B| = |A|+|B|$ A&B are disjoint set $|A \cup B|=|A|+|B|$ Power set has 2^n elements
One to one (injection) the second set has at most 1 arrow pointing to it.
Onto (surjective) the second set has at least 1 arrow pointing to it.
Bijection. Onto and one-to-one exact amount. One points to one other and only one.

Algebraic Proof of Set Identity
For all sets A,B,C $(A \cup B)-C=(A-C) \cup (B-C)$
$(A \cup B)-C = (A \cup B) \cap C^c$ by the set difference law
$=C^c \cap (A \cup B)$ by the commutative law for $\cap$
$=(C^c \cap A) \cup (C^c \cap B)$ by the distributive law
$= (A \cap C^c) \cup (B \cap C^c)$ by the commutative law for $\cap$
$= (A-C) \cup (B-C)$ by the set difference law

For all sets A and B, $A-(A \cap B) = A-B$

$A-(A \cap B) = A \cap (A \cap B)^c$ by the set difference law
$= A \cap (A^c \cup B^c)$ by DeMorgan's law
$= (A \cap A^c) \cup (A \cap B^c)$ by the distributive law
= empty set $\cup (A \cap B^c)$ by the complement law
$= (A \cap B^c) \cup$ empty set by the commutative law for $\cup$
$= A \cap B^c$ by the identity law for $\cup$
= A-B by the set difference law.

Proving Ontoness:
Recall $F:X \rightarrow Y$ is onto if
$\forall y \in Y, \exists x \in X$ s.t. $F(x)=y$
To prove F is onto:
Pick an arbitrary y in Y.
Show $\exists x \in X$ s.t. $F(x)=y$
To prove F is not onto:
Find a y in Y s.t. $y != F(x)$ for any x in X

Proving One-to-oneness
Need to prove: $\forall x_1, x_2 \in X, F(x_1)=F(x_2)$ ➔ $x_1=x_2$
Thus, pick arbitrary elements x_1 and x_2 of X
Assume $f(x_1) = f(x_x)$.
Show that $x_1=x_2$
To show f is not one to one:

Find elements x_1,x_2 in X s.t.f(x_1)=f(x_2) but x_1!=x_2
Inverse functions F^{-1}: Y→X, s.t. ∀y∈Y,F^{-1}(y)=x∈X s.t. F(x)=y
Composition of Functions: Let f: X→Y' and g:Y→Z be functions s.t. y'⊆ *Y*
The composition of f and g is function g°f: X→Z: (g°f)(x) = g(f(x)) for all x∈X